Assignment No.: ______ Page No.: ______
Subject ______
Roll No. ______

NCRD's **Sterling** INSTITUTE OF MANAGEMENT STUDIES, NERUL

1

MAY 2017 → ASSIGNMENT - I

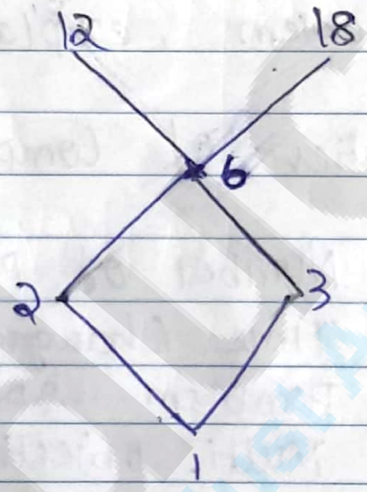
Q1 a) if A = [1, 2, 3, 6, 12, 18] and the partial order relation R is the divides relation i.e a|b aRb iff (a divides b)

i) Draw the Hasse diagram for the poset (A, R)

ii) Find the minimal & maximal element, least & greatest element if exits.

iii) if B = (6, 12, 18) and find all the lower bounds and upper bounds of B & LUB & GLB of B

(i)



ii) minimal element = 1
Maximal element = {12, 18}

least element ~~of~~ = 1
Greatest element = does not exist.

NO

(iii) NO upper bounds of ~~A~~ = {12, 18, 6}, LUB of ~~A~~ = 6
Lower bouds of B = {1, ~~2,3~~}, GLB of B = 6
{1, 2, 3, 6}

Q 1 b) without using truth table prove
$(P \to Q) \wedge (R \to Q) \equiv (P \vee R) \to Q$

$\equiv (\sim P \vee Q) \wedge (\sim R \vee Q)$ implication law
$\equiv (Q \vee \sim P) \wedge (Q \vee \sim R)$ commutative law
$\equiv Q \vee (\sim P \wedge \sim R)$ Distributive law
$\equiv Q \vee (\sim (P \vee R))$ De-Morgan's law
$\equiv \sim (P \vee R) \vee Q$ Commutative law
$\equiv (P \vee R) \to Q$ implication law
$\equiv$ RHS

$\therefore$ Hence proved.

Q.1 c) What are the characteristics of a complex business problem, explain any two

Ans Characteristics of Complex Business Problems:-

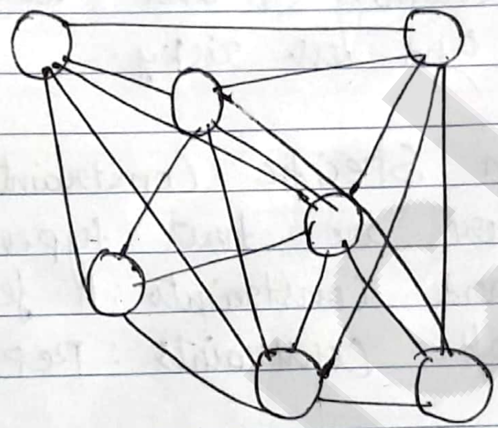
1) Number of possible solution
2) Time changing environment
3) Problem specific constraints
4) Multi objective problems.

The statement "complex business problems are difficult to solve" is so obvious that it does not require any justification.

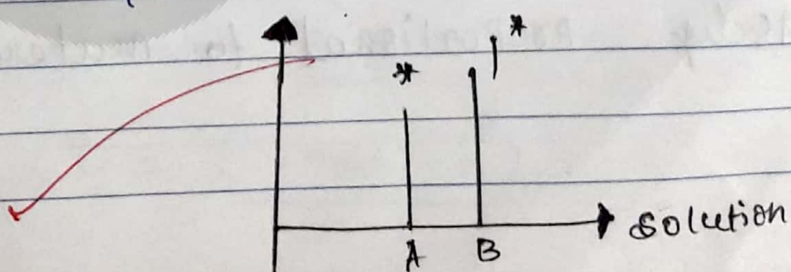
1) Number of possible Solutions:-
=> The number of possible solutions is so large that it precludes a complete search for the best answer.

⇒ For example:- Consider Travelling Salesman Problem: Travelling the shortest possible distance, the salesman visits each and every city in his territory (exactly once) and then returns to the home city.

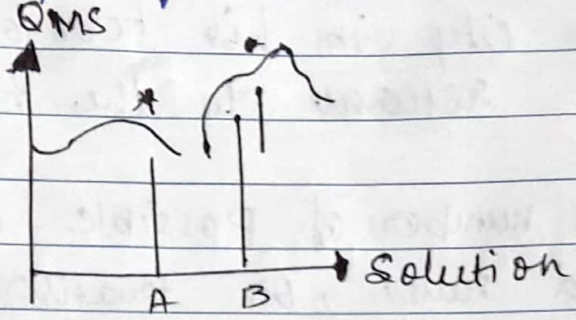
⇒ The number of possible distribution, routes, fraud rules, or transportation plans might be so large that examine all possibilities.



2) Time Changing environment:
⇒ Problem exists in a time changing environment. That means that yesterday's decision, however optimal, may be far from optimal today.
Eg:- Travelling Salesman Problem with many trunks.
⇒ Problem influenced by many environmental factors any solution to static snapshots of this problem proves useless.



Case 1 :- Considering implementation of a solution A or B. Solution B is better than Solution A has higher quality measure score.



Case 2 :- In this condition solution A is better than Solution B and solution A is more stable and less risky.

3) Problem Specific Constraints :-

There are two types of constraints

1) Hard Constraints : A feasible Solution cannot violate
2) Soft Constraints : Represent requirement not mandatory.

Soft constraints eg :- personnel preference.

4) Multi Objective Problems :- There are many (possible conflicting) objectives.

Real world business problem may have more than one objective.

Eg. Objective may include minimization of production time & minimization of material waste etc both are : Production time is inversely proportional to material waste.

Q2 b) Use Mathematical induction to prove the property $P(n)$

$P(n): 3^n + 2n - 1$ is divisible by $4 \; \forall n \in N$

$1 + 2 + 3 + \cdots\cdots + n = 3^n - 2n - 1$

$L.H.S = 1$

$R.H.S = 3^n - 2n - 1$

$= 3 - 2 - 1$

$= 4$

It is divisible by 4

$\therefore P(1)$ is true

Step 2 : Assume $P(k)$ is true for some $k < n$

$\therefore 3^k + 2k - 1$ is divisible by by

$\therefore 3^k + 2k - 1 = 4m$ for some $m \in Z$

Step 3 :- Prove that $P(k+1)$ is true

Consider , $3^{(k+1)} + 2(k+1) - 1$

$= 3^k \cdot 3 + 2k + 2 - 1$

$= 3 \cdot 3^k + 6k - 3 - 4k + k$

$= 3(3k + 2k - 1) - 4\ (k-1)$

$= 3 \times 4m - 4(k-1)$

$= 4(3m - k + 1)$

$= 4m$, $\because$ where $m_1 = 3m - k + 1 \in Z$

$\therefore P(k+1)$ is true , Hence $P(n)$ is true for every

Q.3. a) Use SAW method to determine the best car. The beneficiary criteria are Durability in years and Resale value other are non beneficiary criteria

Type of car	− MAITANCE	− Purchase price in Rs	+ Durability in year	+ Resale value
Car 1	800	350000	6.5	100000
Car 2	1000	1000000	10	450000
Car 3	1250	650000	10	290000
Weight	0.15	0.4	0.25	0.2

Type of car	Maitance	Purchase price in Rs	Durability in year	Resale value
Car 1	1	1	0.65	0.22
Car 2	0.8	0.35	1	1
Car 3	0.64	0.53	1	1.55

Car 1 = $1 \times 0.15 + 1 \times 0.4 + 0.65 \times 0.25 + 1 \times 0.2$

$= 0.15 + 0.4 + 0.1625 + 0.2$

$= 0.9125$ 0.76

Car 2 = $0.8 \times 0.15 + 0.35 \times 0.4 + 1 \times 0.25 + 1 \times 0.2$

$= 0.12 + 0.14 + 0.25 + 0.2 = 0.71$

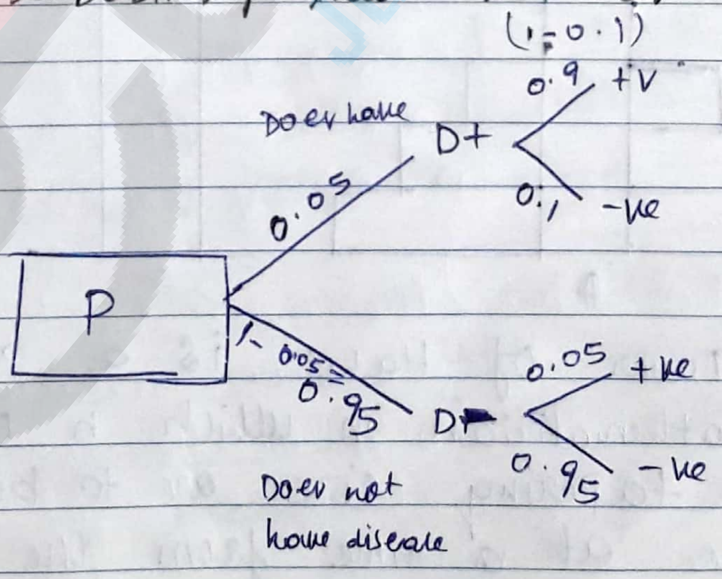
Car 3 = 0.64 x 0.15 + 0.53 x 0.4 + 0.25 x 1 + 0.2 x 1.55

= 0.096 + 0.212 + 0.25 + 0.31

= ~~0.868~~ = 0.69

∴ Car 1 is the best car because it has larger value.

Q3 b) In a screening test for a disease. The frequency of the disease in a population is 0.5%. The test is highly accurate with 5% false positive rate & 10% false negative rate. A person takes the test and it comes positive. Construct a decision tree and use Baye's theorem to determine the probability that he has a disease?



Bayes Therm :-

$$P(D+|T+) = \frac{P(T+|D+) + P(D+)}{P(T+)}$$

$$P(T+) = 0.9 \times 0.05 + 0.05 \times 0.95$$
$$= 0.0925$$

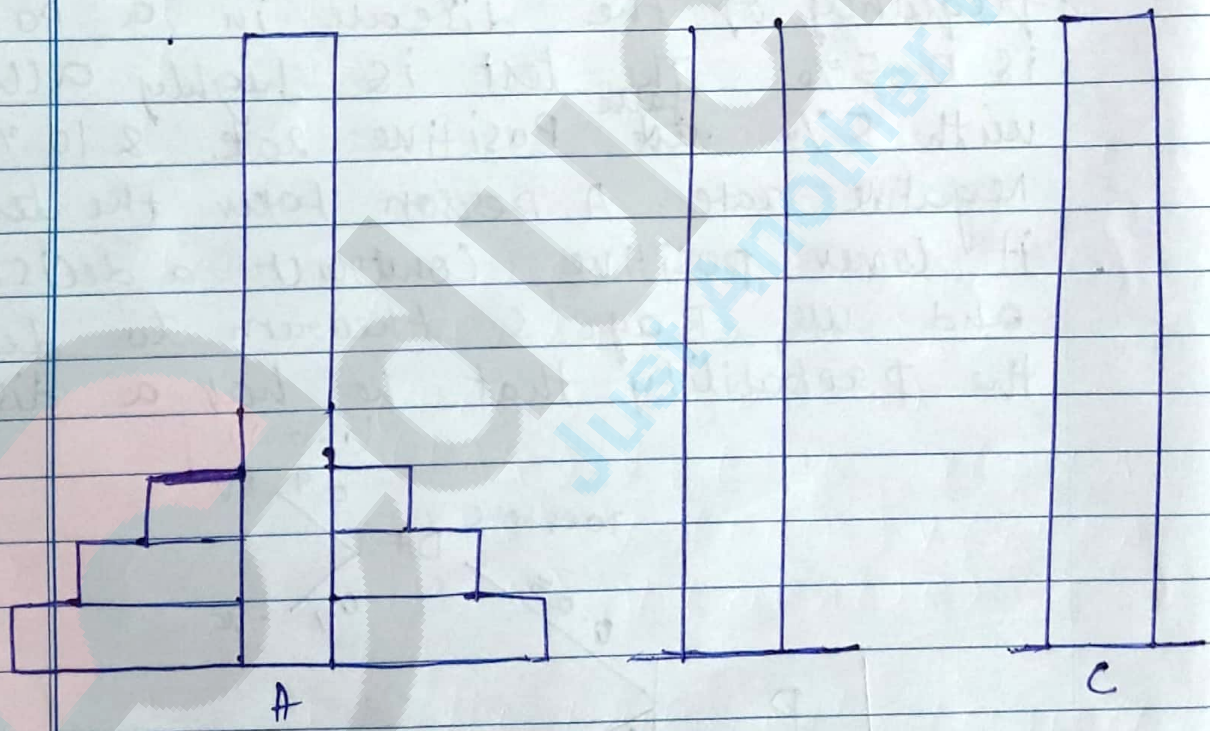
$$P(T+|D+) = 0.9 \times 0.5 = 0.045$$

$$P(D+) = 0.05$$

$$P(D+|T+) = \frac{P(T+|D+) \times P(D+)}{P(T+)}$$

0.082949 $= \frac{0.035 \times 0.05}{0.0925} \Rightarrow 0.02432$

Q4) State the "Tower's of Hanoi problem:-



Tower of Hanoi is a puzzle by mathematician in which n circular rings of tapering size are to be transferred one at a time from one tower to another and there is a third tower available on which rings can be left temporarily.

If during the course of transferring the rings no ring may ever be placed on the top of a smaller one. The problem is to find in how many moves can these rings can be transferred with their relative positions unchanged.

Sol:- Let T_n be the minimum no. of steps needed to move n-rings from one tower to another

$$T_{n-1} + T_{n-1} + 1$$
$$T_n = 2T_{n-1} + 1$$

The top n-1 rings from 1^{st} tower are to be moved to the third tower recursively that is done in T_{n-1} moves.

We require to move the last ring into the 2^{nd} tower. Now move n-1 rings from the

3^{rd} tower recursively to the end tower which will again require T_{n-1} moves. This shows that n rings can be transferred, the eqn will be

$$T_n = 2T_{n-1} + 1$$
$$T_n = 2(T_{n-2} + 1) + 1$$ Backtracking method

$$T_n = 2^2 \; T_{n-2} + 2 + 1$$

$$T_n = 2^2 (2 \, T_{n-3} + 1) + 2 + 1$$

$$T_n = 2^3 \; T_{n-3} + 2^2 + 2 + 1$$

Sub it r times

$$T_n = 2^r \; T_{n-r} + 2^{r-1} + \dots + 2 + 1$$

$$= 2^r \; T_{n-r} + \frac{2^r - 1}{2-1}$$ sum of G. P

Substituting $r = n-1$

$$T_n = 2^{n-1} \; T_{n-(n-1)} + 2^{n-1} - 1$$

$$T_n = 2^{n-1} \; T_1 + 2^{n-1} - 1$$

$$\boxed{T_n = 2^n - 1}$$

Q.4 ~~Xon~~ write the truth table for α & find
b) the principal CNF & principal DNF
of α

$$\alpha = (\sim P \vee \sim Q) \rightarrow (P \leftrightarrow Q)$$

Ans Truth table :-

P	Q	$\sim P$	$\sim Q$	$\sim P \vee \sim Q$	$P \leftrightarrow Q$	α
T	T	F	F	F	T	T
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

Principal Disjunction Normal form :-

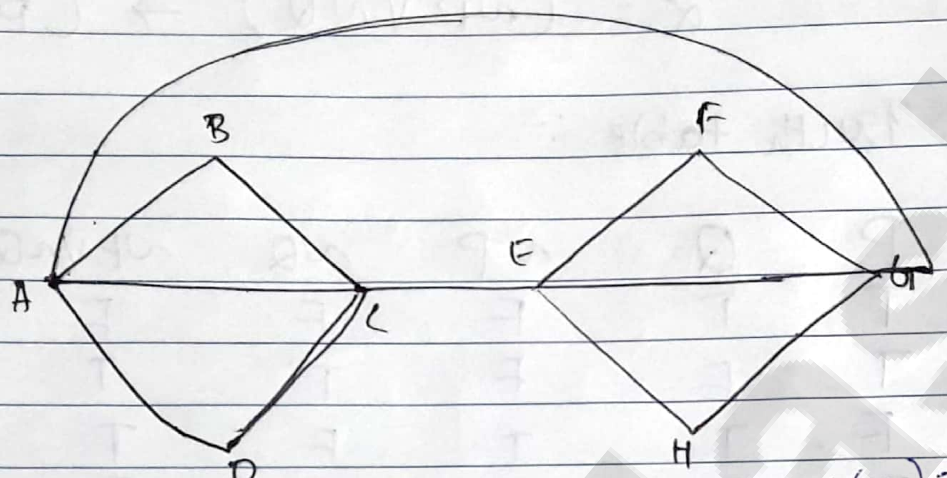
$$(\sim P \vee \sim Q) \rightarrow (P \leftrightarrow Q)$$
$$= (P \wedge Q) \vee (P \leftrightarrow Q)$$
$$= (P \wedge Q) \vee [(P \wedge Q) \vee (\sim P \wedge \sim Q)]$$
$$= (P \wedge Q) \vee (P \wedge Q) \vee (\sim P \wedge \sim Q)$$
$$= (P \wedge Q) \vee (\sim P \wedge \sim Q)$$

To find the principal conjuctive Normal form consider the remaining form in principal Disjunctive normal form. These forms are : $(P \wedge \sim Q) \vee (\sim P \wedge Q)$
Take the negotion of this we get the required principal conjuctive normal form.

Hence the principal conjunctive Normal form is $(\sim P \vee Q) \wedge (P \vee \sim Q)$

Q.5 Find the Euler Path & Euler Circuit in the following graph if they exist

1)

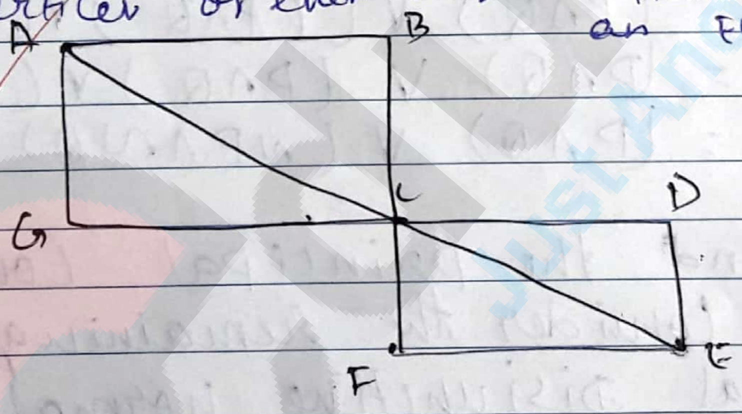


Euler path :- Vertex (A) = 4 V(B) = 2
~~Euler~~ V(C) = 4 V(D) = 2 V(E) = 4
V(F) = 2 V(G) = 4 V(H) = 2

Euler circuit :- B → A → G → F → E → H → G → E → C → D → A → C → B

This circuit consist of all vertices or even degree. Hence there can be an Euler circuit

2)



Euler path :- V(A) = 3 V(D) = 2
V(B) = 2 V(E) = 3 V(C) = 6 V(F) = 2

Euler circuit :- A → B → C → D → E → F → C → G → A → C → E

In this graph there are 2 vertices of odd degree there is no Euler circuit.

Assignment No.: Page No.:
Subject :
Roll No.:

Q5) b) find the recurrance relation of the ~~Recurrance rela~~ solution.
$C_0a_n + C_1a_{n-1} + C_2a_{n-2} = f(n) = 40$
is $2^n + 3^n + 5$, find C_0, C_1, C_2

$\Rightarrow C_0a_n + C_1a_{n-1} + C_2a_{n-2} = f(n)$

$\Rightarrow$

$a_n = 2^n + 3^n + 5$
$a_{n-1} = 2^{n-1} + 3^{n-1} + 5$
$a_{n-2} = 2^{n-2} + 3^{n-2} + 5$

$C_0a_n + C_1a_n + C_2a_{n-2} = f(n)$

$\Rightarrow C_0(2^n + 3^n + 5) + C_1(2^{n-1} + 3^{n-1} + 5) + C_2(2^{n-2} + 3^{n-2} + 5) = 40$

$\Rightarrow C_02^n + C_03^n + 5C_0 + C_1(2^n.2^{-1}) + C_1(3^n.3^{-1}) + 5C_1 + C_2(2^n.2^{-2}) + C_2(3^n.3^{-2}) + 5C_2$

$\Rightarrow C_02^n + C_03^n + 5C_0 + C_1\left(\frac{2^n}{2}\right) + C_1\left(\frac{3^n}{3}\right) + 5C_1 + C_2\left(2^n.\frac{1}{4}\right) + C_2.3^n\left(\frac{1}{9}\right) + 5C_2 = 40$

Taking 2^n & 3^n

$$2^n\left(C_0 + \frac{C_1}{2} + \frac{C_2}{4}\right) + 3^n\left(C_0 + \frac{C_1}{3} + \frac{C_2}{9}\right) + (5C_0 + 5C_1 + 5C_2) = 40$$

$\Rightarrow C_0 + \frac{C_1}{2} + \frac{C_2}{4} = 0 \longrightarrow (1)$

$\Rightarrow C_0 + \frac{C_1}{2} + \frac{C_2}{9} = 0 \longrightarrow (2)$

$5C_0 + 5C_1 + 5C_2 = 40 \longrightarrow (3)$

$C_0 + C_1 + C_2 = \frac{40}{8} = 8$

eq ① $c_0 + \frac{c_1}{2} + \frac{c_2}{4} = 0$

$4c_0 + 2c_1 + c_2 = 0$

eq ② $c_0 + \frac{c_1}{3} + \frac{c_2}{9} = 0$

$9c_0 + 3c_1 + c_2 = 0$

eq ③ $c_0 + c_1 + c_2 = 8$

$4c_0 + 2c_1 + c_2 = 0$

$-9c_0 + 3c_1 + c_2 = 0$

$-5c_0 - c_1 = 0$

$5c_0 + c_1 = 0$ —— ④

$8c_0 + 2c_1 = -8$ ⟶ ⑤

$4c_0 + c_1 = -4$

$5c_0 + c_1 = 0$

$-c_0 = -4$

$\boxed{c_0 = 4}$

$4c_0 + c_1 = -4$

$4(4) + c_1 = -4$

$16 + c_1 = -4$

$c_1 = -4 - 16$

$\boxed{c_1 = -20}$

eq. ③

$c_0 + c_1 + c_2 = 8$

$4 - 20 + c_2 = 8$

$-16 + c_2 = 8$

$c_2 = 8 + 16$

$\boxed{c_2 = 24}$

$$M_R \infty = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

Q6 (4) A is a set of Real ∞ & aR_b iff $|a-b| \le 2$

Ans 1) R is reflective because $|a-a| \le 2 \ \forall a \in R$

eg $(a,a) = (3,3)$

$|3-3| \le 2$

ii) R is not irreflective b/z $|3-3| \le 2$ for $3 \in A$

iii) R is symmetric becoz for $|a-b| \le 2$

$aRb \to bRa$

eg. $(1R2) \to (2R1)$

$|(1-2)| \Rightarrow |(2-1)|$

$= 1 \le 2 \Rightarrow 1 \le 2$

iv) R is not assymetric because for $|4-3| \le 2$ we have $|3-4| \le 2$ i.e $4R3 \to 3R4$

v) R is not antisymmetric becoz $4R3$ & $3R4$ but $4 \neq 3$

6a) $A = \{a, b, c, d, e\}$, $R = \{(a,a), (a,d) (c,e) (d,a) (c,d) (e,c)\}$
Determine the R^n relation using warshall's algorithm.

$$W_0 = M_X = \begin{array}{c|c} & \begin{matrix} a & b & c & d & e \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \\ e \end{matrix} & \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$N = 4$

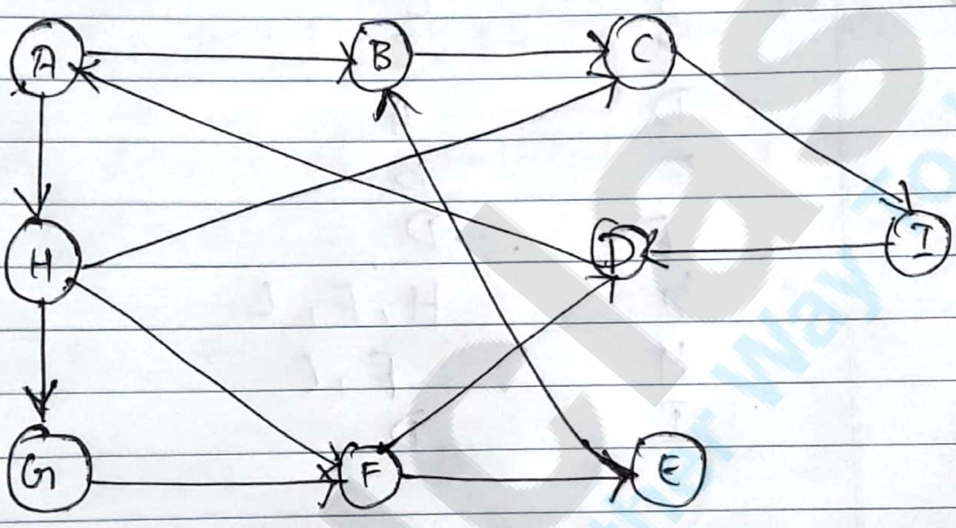
$$W_1 \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad W_3 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$W_2 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \qquad W_4 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$W_5 = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

vi) R is not transitive because 4R3 & 3R1 but 4R1 because |4-1| ≤ 2

Q6 d) Find Adjacency Matrix & Adjacency list



i) Adjacency matrix :-

Matrix =

$$
\begin{array}{c|ccccccccc|}
 & A & B & C & D & E & F & G & H & I \\
\hline
A & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
B & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
C & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
D & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
E & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
F & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
G & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
H & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
I & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{array}
$$

Adjacency list :-

Vertex :-

A	B
B	C
C	I
D	A
E	B
F	D
G	H, F, C
H	F, C
I	D.